Analysis and Modeling of Southern California Deformation

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Abstract

The large and rich quantity of data now becoming available through the Southern California Integrated GPS Network (SCIGN) makes possible a wide variety of research involving analysis and modeling. The Hector Mine earthquake (M=7.1) is the outstanding geophysical event so far registered by this network in its mature form, with 34 stations registering permanent deformation at the 95% confidence level. Inversion of this data for fault slip parameters was possible immediately after the event, and resulted in a pure GPS solution for the fault location, slip, and orientation, which have held up well under subsequent scrutiny. A new high-precision re-analysis of the SCIGN data has provided data sets for 71 stations with over one year's high-quality data. This data set has been subjected to principal components analysis, resulting in some marginal improvements in understanding at this preliminary stage. Finite element analysis in two and three dimensions is another important technique for explaining such network data, for example in comparing different hypothetical faulting structures and rheologies in the Los Angeles Basin. Automatic mesh generation based on geophysical fault and layering description is an important technology under development to assist the geophysical modeler.

Introduction

An appropriate goal for the geodetic community in the coming decade is a suite of integrated models that can be incrementally adjusted by all emerging data. But what we have at present is a burgeoning supply of space-geodetic data with no adequate modeling framework beyond slow hypothesis testing. The transition will necessarily require a time of bolstering the old methodology with better tools. We require larger, more sophisticated models, but also simple, fast models with high data throughput and non-physics-based techniques that find clusters of interesting activity. Therefore we exploit concurrently a broad range of data types, a high-performance finite element model, a half-space dislocation model and its inverse, and pattern recognition techniques. This heterogeneity is challenging, but enables creative testing with feedback and iterative correction of ideas. Our near-term goal is establishing the parametric description of the North American plate boundary and its earthquake cycles. This should provide a sound basis for the long-term, where we seek a systems description of faults and their stress environments with formal error budgets, and automated data assimilation.

We present some of the current work, illustrating several of these modeling and pattern analysis techniques, each exploiting data from the Southern California Integrated GPS Network (SCIGN). First we present use of an iterative half-space dislocation inversion technique for rapid estimatial inversion is in [1]; here we discuss robustness of the solution and prospects for automatic fast turn-around and availability of results for future events. Second we show an example of non-physics-based pattern extraction, the use of blind PCA analysis on a collection of the SCIGN data time series, and the post-hoc insight into the physical system that are gained thereby. Finally we present some work in progress in finite element modeling and mesh generation, focusing on their utility as tools that enrich a community effort at data interpretation.

Pure GPS estimate of the Hector Mine earthquake dislocation

The inversion code used for the result in [1] is conceptually simple. It combines the simulated-annealing downhill simplex method of [2] with the Okada model [3] for surface dislocation in an elastic half space. We have further added a technique for estimating the curvature matrix for chi-square with respect to model parameters at the solution point, which provides an estimate for the formal errors on the parameter estimates. Also we have designed the technique to be widely useful by allowing many observation types. These include geodetic position, relative position with respect to a particular station, line-of-sight displacement to a satellite (as required for interferometric SAR), strain, and pseudo-observations defined as a-priori constraints on the fault and fault slip model parameters.

One might well wonder if the estimate of fault parameters is robust, since the Hector event occurred well to the east of the SCIGN network, which is designed to evaluate hazard to the Los Angeles area, nearly 200 km to the southwest. Two observations bear on this issue. The first also demonstrates the timeliness of our estimated results. As may be seen from http://milhouse.jpl.nasa.gov/hector, we posted four estimates on four succeeding dates. These differ chiefly in the quality of the GPS estimate of coseismic slip and the particular list of stations available for analysis at those times, all very soon after the Hector event. The first set (posted October 19, three days after the earthquake) involved a mixture of GPS processing types, some with and some without attempts to resolve phase bias in the GPS carrier signals. The dislocations are estimated from the day-averaged positions the day before and day after the event. The reference frame was inadequately determined, so the inversion code used displacements relative to the station GOL2 (Goldstone, California). In full, 67 stations were used. The second solution is similar, but uses a free-network solution (no station was considered fixed), a set of 76 stations were used with phase-biases estimated. This solution was posted October 22. Then a third solution was posted October 25 involving 47 stations. This set of displacements used advanced techniques for determining the reference frame and superior phase-bias estimates for each station. Also the coseismic displacement estimate uses five days before the event and four days after. These first three solutions estimated the location, strike, length, width, and strike-slip of the event, assuming a rectangular dislocation including the surface on a vertical fault segment. The latter data set was also used to estimate dip and dip-slip, in an extension of the third model.

We find remarkable the consistency of these four solutions. These imply event moment between 3.65 and 4.04 x 10^{26} dyne-cm, only a 10 % spread. These are all consistent with M=7.0, essentially matching the seismic moment of 7.1. All four show surface rupture in close agreement with the imaged rupture on geodetic-registered photos; for example the strike estimates are –26, –28, –26, and –29 degrees from north. Estimates of width, length, and strike-slip show more spread (up to 70%), each solution picking a different trade-off consistent with the location and magnitude.

Initial attempts at solutions used iterative starting points generally consistent with early seismic and geologic reports. Some of these were only representative values based on local measurements, for example we used an initial strike of –10 characteristic of early reports from near the epicenter. Iterative solutions stubbornly insisted on strikes more like –28, which turned out to be more representative of the average of the Hector surface rupture, suggesting a reasonable degree of robustness for the technique.

More impressive evidence of robustness is found in picking an initial rupture location 150 km away from the fault in various directions. Most of these converged on near-identical solutions to those found from close initial guesses; those that did not converged on some shallow local minimum of chi-square nowhere near the true Hector rupture. These tests present a picture of a robust technique, that can be trusted to process GPS data in an automated mode to improve on a seismic estimate of initial location, initially lacking other fault model parameters. This confidence can be further enhanced by applying a test of reasonable fit to the converged solution. Our good solutions had chi-square per DOF in the range 1.2-6 (with inverse dependence on data uncertainty), while the infrequent false solutions from wildly far initial conditions displayed final chi-square per DOF of many hundreds, and chi-square curvature matrices that were not positive definite. Based on this single event at the sparse boundary of the SCIGN network, it appears coverage does not need to highly overlap the central deformation zone to obtain a good estimate of rupture as a single-fault model.

Principal components in SCIGN data

A reanalysis of SCIGN data to produce time series of station positions is described at http://milhouse.jpl.nasa.gov/scign/analysis/description_2.0.0.html. Some of the improvements are regional estimation of phase bias cycles, and definition of a local realization of the ITRF97 reference frame. These improvements reduce the daily repeatability (1-sigma) from 5.5 to 1.3 mm in the horizontal direction. Time series available at this writing span from January 1998 to April 2000. We chose 71 stations (of 127) that have high-quality data for more than 1 year. As each has 3 coordinates this provides a rich set of 213 time series suitable for principal components (PCA) analysis.

One challenge is that the data is not uniform in formal uncertainty; in particular the vertical series have estimated scatter 3x larger than the horizontal. We scaled each full time series by its average uncertainty before performing the PCA analysis, and convert back to cm for analyzing results.

Another issue is gaps in the data. We devised two ad-hoc approaches for this, first doing linear interpolation for series gaps less than 4 days long, and second relying on an iterative PCA reconstruction of the missing data, described below.

PCA analysis is an application of singular value decomposition. The N_s time series are placed in the columns of an N_t x N_s array **A**. Standard techniques (for example many commercial math and visualization packages) derive a factorization $\mathbf{A} = \mathbf{UWV}^T$ with the properties that **U** has the same dimensions as A and columns that are orthonormal, **W** is a diagonal square matrix of order N_s, and V is N_s x N_s with orthonormal columns. Columns of U, W, and V may be reordered so that the elements of **W**, w_p, appear in descending order of size (they are positive). One may consider truncating **U, W, V** to the first p columns, zeroing the rest, and multiplying to reconstruct a reduced version of $\mathbf{A}$, $\mathbf{A_p}$. It can be seen that each such approximation is closer in L2 norm to $\mathbf{A}$ than the ones with smaller p. When the values of w_p fall off rapidly the approximation may be quite good for even small values of p.

Missing data is initially set to zero, but allowed to change; all other data is fixed through the iterative fill-in technique. This initial $\mathbf{A}$ is used to find $\mathbf{U}$, $\mathbf{W}$, $\mathbf{V}$, from which is retained but the first columns, from which we construct $\mathbf{A_1}$. Values of $\mathbf{A_1}$in positions corresponding to missing data is copied to $\mathbf{A}$, and the process repeated a few times. Then in similar fashion we find $\mathbf{A_2}$, and further refine the missing values for a few more iterations. This is followed up to $\mathbf{A_8}$, where we terminate the process due to observation that the changes are slight.

At this point we consider the full decomposition. For any column index k we may form the rank-one combination $w_k\ \mathbf{U_k}\ \mathbf{V_k}^\mathbf{T}$ and consider that "mode k." Note it has a temporal signature $\mathbf{U_k}$, a spatial signature $\mathbf{V_k}^\mathbf{T}$, and amplitude w_k. Because this data set spans the time and space of the Hector Mine earthquake, it is not surprising to find the first two modes are highly correlated with the spatial signature of the Hector displacement. Both have time signatures containing a jump on October 16, 1999, together with the background secular variation. These two modes (that is, $\mathbf{A_2}$) account for 98.5% of the variance in the entire set of (error-scaled) time series. Mode 3 displays a roughly sinusoidal annual time signature, with a spatial pattern that shows little correlation with station distance. It may represent errors in the GPS tidal or ocean-loading models, or the signature of annual water table variations. There are several additional cryptic modes at seasonal to annual time scales, then a cluster of several modes showing a coherent time signature with period 13.6 days, probably due to GPS ocean-loading model errors. More questionable are much smaller possibly coherent signals at 3-8 day periods, which could represent local weather or GPS satellite orbit model errors.

We do not see any modes resembling reference frame noise, for example with a correlated spatial signature and noise-like time history. This may indicate the high quality of the regional ITRF97 technique, which was used in the SCIGN 2.0 analysis.

Most of the modes appear to show no correlation in time or space, and so may be considered noise. This suggests we may choose a reasonable value of p, such that $\mathbf{A_p}$ represents a filtered version of $\mathbf{A}$, and hence produce a filtered version of each station time series. This approach has clear advantages to filtering individual stations separately, as it keeps presumably geophysical signals that are found at multiple stations, while removing true noise and unique single-station effects.

Mesh generation and finite elements

Methods of pattern detection may be used to attempt isolation of physical causes through inversion models such as we have outlined above. We suppose from experience that many underground causes of surface deformation may be roughly modeled as simple sources, such as fault slip or Moghi sources, and so may be reasonably well localized and even distinguished by fast methods with coarse approximations as we demonstrated for the Hector Mine estimates. These rough pictures may be refined by more refined models, which may be too cumbersome for nonlinear inverse methods but may yield refinement by running a suite of models and making careful comparisons. We have developed a general faulted viscoelastic finite element model to aid in such analysis.

Experience shows that a major roadblock to finite element analysis for moderately complex geometry is the generation of a good mesh. Such a mesh must faithfully represents the geometry of the faults and other boundaries, have no elements with high aspect ratio, and is also numerically accurate in the regions of high displacement and stress gradients, which may be difficult to predict a priori. Many analysts can testify to the steep learning curve of commercial and other mesh generation packages, and the difficulty of producing acceptable meshes. Geophysical researchers can not, as a rule, spend a large amount of time doing finite element analysis; it is but an occasional pursuit. This indicates the desirability of having a specialized finite element interface tailored to their needs. We have been working with Professor Jin-fa Lee of the Ohio State University, who has co-authored academic and commercial mesh generators. The work is still in progress, but the aim is to make the mesh generation oriented to geophysics. For example faults are specified according the familiar parameters length, width, strike, dip and so on, which the user may directly transfer from a database. Generation is speeded by automatic iteration, including one or more test solutions using an initial course grid to indicate regions requiring additional meshing.

The finite element code itself strives for general use. The volume is defined by space-filling tetrahedra, which readily conform to complex geometry. Displacement at nodes is the fundamental variable; stress within elements is derived from that. Surface nodes carry special conditions such as imposed displacements or tractions. This software can solve elastostatic problems as well as time-marching faulted viscoelastic problems. Faults can fail at specified times or conform to a fault failure strength model or other physical models. Fault slip is accounted for with split nodes; neither the faults nor other mesh features deform during solution. Materials may be elastic or viscoelastic, and viscosity may be either of linear or power-law type. Material properties may be assigned arbitrarily to each element, which supports both layering and horizontal variation.

Conclusion

In this paper we have composed brief reports of use and design of software tools for modeling surface deformation data, which is accumulating in the geophysical community at a rapid rate. We have tried to show how highly approximate models such as used for the Hector Mine inversion, and even physics-ignorant "models" such as principal components analysis work together in natural partnership and with finite element techniques. We seek to go beyond hypothesis testing and find ways to use these tools synergistically and creatively under the stimulus of special geophysical events and opportunities, to develop a comprehensive model for Southern California deformation that can reap the full benefit of space-geodetic and other expanding data sets.

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